The MoSST_DAS SV forecast model for 2010-2015

The MoSST_DAS SV forecast model for 2010-2015 is derived from geomagnetic data assimilation in which the numerical dynamo model (MoSST core dynamics model) is integrated with several geomagnetic field models over the past 7000 years. The specific parameters and algorithms in the SV forecast model are summarized as follows. We refer the reader to the details in the cited references.

1. Field models

Four different field models are used to generate Gauss coefficients at the mean surface of the Earth for the geomagnetic data assimilation. They are

CALS7K-2 provides the coefficients for the period from 5000BCE to 1620.0.

Gufm1 provides the coefficients for the period from 1650.0 to 1960.0.

CM4 provides the coefficients for the period from 1965.0 to 2000.0.

CHAOS-2s provides the coefficients from 2001.0 to 2009.0.

In the period from 1620 to 1650, the coefficients change smoothly from CALS7K to gufm1; in the period from 1960 to 1965, the coefficients change smoothly from gufm1 model to CM4; and in the period from 2000 to 2001, the coefficients change smoothly from Cm4 to CHAOS.

The Gauss coefficients from the field models are then downward continued to the top of the D"-layer, and are then assimilated into the MoSST numerical model solutions for the forecast. In the calculations leading to the forecast SV model, only the coefficients of the first eight degrees $L \leq 8$ are used in assimilation.

2. MoSST_DAS

MoSST_DAS is the geomagnetic data assimilation system (framework) used for the forecast. The dynamics of the magnetic field time variation is modeled by the MoSST core dynamics model. The assimilation algorithm used in this SV forecast is the optimal interpolation (OI) algorithm (Kuang et al 2008,; Kuang et al 2009).

The parameters used in the MoSST core dynamics model include the Rayleigh number R_{th}, the magnetic Rossby number R_o, the Ekman number E and the modified Prandtl number q_κ. The specific values of the parameters are

$$R_{th} = 15000, \quad R_o = E = 1.25 \times 10^{-6}, \quad q_\kappa = 1.$$

There is an electrically conducting D"-layer above the core-mantle boundary (CMB). Its thickness is 20km, and its electrical conductivity is 1/20th of the core fluid electrical conductivity σ which is defined such that $\mu\sigma r_{cmb}^2 = 200000$ years (r_{cmb} is the mean radius of the CMB). In the simulation, "free-slip" and "fixed heat flux" boundary conditions are used. The numerical solutions are presented by the spherical harmonic expansions for each radial grid point. And the spherical harmonic coefficients of the expansion are approximated by the values on the radial grid points. The truncation orders are 44×44×90. For the details of the MoSST model, we refer the reader to Kuang and Bloxham (1999), Kuang and Chao 2003, Kuang et al (2008) and Jiang and Kuang (2009).

It should be pointed out here that in MoSST_DAS, observations (i.e. the field model output) are assumed accurate, while the errors are primarily due the difference between the numerical model and the Earth's outer core. This is primarily due to the facts that (1) there is no error estimation on the individual Gauss coefficients from the field models and (2) the model errors are expected to be much larger than those from observations. If observation errors are provided in the field model output, they can be promptly included in assimilation.

3. Calculation of the SV forecast

In MoSST_DAS, the ratio of the Gauss coefficients are predicted, i.e., for any given degree *n* and order *m*,

$$\tilde{g}_n^m \equiv \frac{g_n^m}{g_1^0}, \quad \tilde{h}_n^m \equiv \frac{h_n^m}{g_1^0} \tag{1}$$

are assimilated into the model solutions. The axial dipole g_1^0 is therefore not forecasted. To calculate the averaged SV over the period (t_0, $t_0+\Delta t$), we use

$$Sg_n^m = \frac{1}{\Delta t}\left[g_n^m(t_0+\Delta t) - g_n^m(t_0)\right] \tag{2}$$

By (1) and (2), we have

$$Sg_n^m = g_1^0(t_0)S\tilde{g}_n^m + \tilde{g}_n^m(t_0)Sg_1^0 + \Delta t Sg_1^0 S\tilde{g}_n^m, \tag{3}$$

where $S\tilde{g}_n^m$ are defined similarly as in (2), except the Gauss coefficients in (2) are replaced by the ratios of the coefficients defined in (1).

For the IGRF SV forecast model, MoSST_DAS provides the forecast of $\tilde{g}_n^m$ in 2010, and their averaged SV $S\tilde{g}_n^m$ for the period 2010-2015. Therefore the axial dipole g_1^0 in 2010 and the averaged SV of the axial dipole Sg_1^0 for 2010-2015 have to be estimated separately.

4. Estimation of the Axial Dipole

The axial dipole g_1^0 in 2010 and the averaged SV of the axial dipole Sg_1^0 for 2010-2015 are estimated using the CHAOS model output for the period 2001 to 2009. The former is estimated through the poly fit extrapolation (see Figure 1), and the latter is assumed simply the mean SV of g_1^0 from 2001 to 2009 (see Figure 2)

5. Error Estimation

The uncertainties in the SV forecast model is based on the following error estimations. The uncertainties for g_1^0 and Sg_1^0 are estimated by the standard deviation of the fit (or average) for the period 2001-2009. The error bound for the forecast of the ratio $\tilde{g}_n^m$ in 2010 and the 5-year SV $S\tilde{g}_n^m$ for 2010-2015 are calculated based on the discrepancies between the 7-year MoSST_DAS forecast and the CHAOS model result from 2002 to 2009. These individual error estimates are then added to (3) to obtain the uncertainties for Sg_n^m of degree *n* and order *m*.

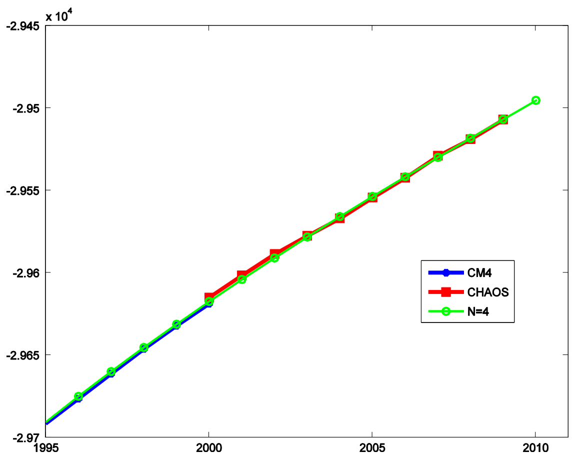

Figure 1: The poly fit extrapolation of the axial dipole coefficient g_1^0 to 2010.

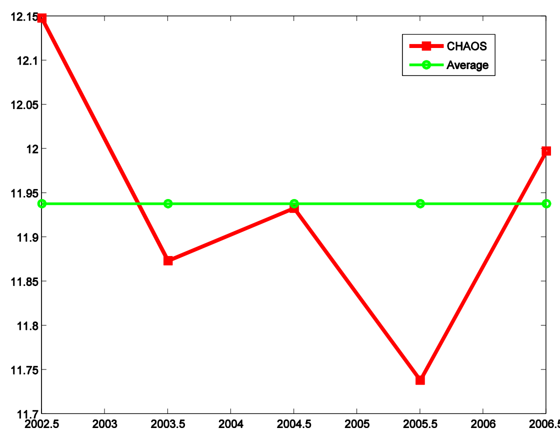

Figure 2: The mean 5-year SV of g_1^0 for the period 2001-2009 from CHAOS model.